Mohammed V University.
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Master of Matter and Radiation.

ADVANCED STATISTICAL PHYSICS

Homework #1 - Fundamendal Concepts

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STATISTICAL THERMODYNAMICS

EXERCISE 1 -ENTROPY- Suppose that a systems A is placed into thermal contact with a heat reservoir A' at an absolute temperature T'. In this process, system A absorbs an amount of heat Q.
Show that the entropy of the system A increases and that it satisfies the inequality $\Delta S \geq Q/T'$.

EXERCISE 2 -THERMODYNAMICS THROUGH STATISTICAL RELATIONS- Consider a system consisting of N_1 molecules of type 1 and N_2 molecules of type 2 confined within a box of volume V. Suppose weak interaction between the molecules so that they consist an idea gas mixture.

- Using a classical approach, how does the total number of states $\Omega(E)$ in the range between E and $E + \delta E$ depend on the volume V of this system ?
- Find the mean pressure $\bar{p}$ as a function of V and T.

STATISTICAL MECHANICS

EXERCISE 3-PARTICLE IN A BOX- Consider a free particle having a mass m and zero spin $s = 0$, placed in a box with side L. The potential energy $U = 0$ inside the box and is infinite outside the box. the energy levels of this system are given by :

$$\epsilon_{n_x,n_y,n_z} = \frac{\pi^2\hbar^2}{2mL^2}(n_x^2, n_y^2, n_z^2)$$

- Calculate the partition function of the particle.
- Find the mean energy $\bar{E}$, the specific heat C_V, the free energy F and the entropy S.

EXERCISE 4 -GIBBS PARADOX- Consider a gas containing N identical monoatomic molecules of mass m enclosed in a container of volume V. The total energy of the gas is :

$$E = \sum_{i=1}^{N} \frac{p_i^2}{2m} + U(r_1, \dots, r_N),$$

where (r_i, p_i) denotes respectively the position and the momentum of the ith molecule. U represents the potential energy of interaction between the molecules.

- For the case of an ideal gas, calculate the partition function Z of a single molecule and that of the whole gas.
- Calculate the mean pressure $\bar{p}$, the mean energy $\bar{E}$ and the specific heat C_V of the gas.
- Find the entropy S of the gas.
- Discuss the results at the level of the second law of thermodynamics.

EXERCISE 5-THE EQUIPARTITION THEOREM- Suppose that the energy of a system is a function of some f generalized coordinates and momentum (q_k, p_k) respectively; i.e,

$$E = E(q_1, \dots, q_f, p_1, \dots, p_f).$$

Where are interested in the case where :

1. The total energy splits additively into the form

   $$E = \epsilon_i(p_i) + E'(q_1, \dots, p_f),$$

   where ϵ_i involves only the one variable p_i and the remaining part of E' does not depend on p_i.

2. The function ϵ_i is quadratic in p_i;i.e, it is of the form

   $$\epsilon_i = bp_i^2,$$

   where b is a constant.

- What is the mean value of ϵ_i in thermal equilibrium if conditions (1) and (2) are satisfied ?

QUANTUM STATISTICS

EXERCISE 6 -SPECIFIC HEAT OF SOLIDS- Consider a solid with Avogadro's number N of atoms per mole. These atoms are free to vibrate about their equilibrium positions, such a movement can be approximated by the behavior of a harmonic oscillator. According to quantum mechanics, the vibration energy ϵ_n is :

$$\epsilon_n = (n_x + n_y + n_z + \frac{3}{2})\hbar\omega$$

- Calculate the partition function of a single atom and of the whole solid.
- Calculate the mean energy $\bar{E}$ and the specific heat C_V of the solid.
- Discuss the limitation of the results.