Mohammed V University.
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Master of Matter and Radiation.

ADVANCED STATISTICAL PHYSICS

Homework #2 - Phase Transitions and Critical Phenomena

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CRITICAL EXPONENTS

EXERCISE 1 -DEFINITION OF CRITICAL EXPONENTS- The critical exponent is a quantity defined as a limiting power law as $T \to T_c$. Take a function $F(t)$ which shows a singularity at $t = 0$, that is :

$$F(t) = A|t|^{\lambda}\left[1 + at^{\lambda_1} + ...\right] \text{ with } \lambda_1 > 0,$$

the critical exponent λ can be extracted by taking the limit :

$$\lambda = \lim_{t\to 0} \frac{\log|F(t)|}{\log|t|}$$

Determine the critical exponent λ of the following functions :

1. $F(t) = at + bt^{1/2} + ct^{1/4}$,
2. $F(t) = at^{-2/3}(t + b)^{2/3}$,
3. $F(t) = at^2 e^{-t}$,
4. $F(t) = at\log(|t|) + b$,

as $t \to 0$, where a, b and c are positive constants

EXERCISE 2 -MAGNETIC SYSTEMS- Consider an equation of state

$$H = aM(t + bM^2) \text{ for } a, b > 0$$

Find the exponents β, γ and δ and verify the scaling relation among them.

MODELS & MEAN FIELD THEORY

EXERCISE 3- THE ISING MODEL- Consider a spin-$\frac{1}{2}$ Ising system on a two dimensional square lattice in presence of an external magnetic field H and nearest neighbor interaction $J(> 0)$. (a) Consider a pair of nearest neighbor spins and (b) consider a four spin cluster in the effective field. Obtain the self-consistent equation for the magnetization associated with one of the spins in both the cases. Calculate T_c in these cases and compare with the mean field as well as with the exact results.